REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
February 20, 1996

3. REPORT TYPE AND DATES COVERED
Final 12/91-8/95

4. TITLE AND SUBTITLE
Mathematical Models of Non-Linear Mechanical and Electrical Systems and Their Qualitative Behavior

5. FUNDING NUMBERS
Grant No.F49620-92-J-0049
Project Task 2364/BS

6. AUTHOR(S)
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AFOSR-TR-96
0219

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9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
Bolling Air Force Base, DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

12a. DISTRIBUTION/AVAILABILITY STATEMENT
unlimited

12b. DISTRIBUTION CODE

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Under this grant my students and I conducted research in the following areas. (1) Nonlinear dynamics of coupled and forced oscillators modelling electric circuits; (2) geometric phases in mechanics; (3) two industrial problems, one in fluid dynamics and the other in granular flow; (4) new geometrical results on parametric resonance; (5) Stability and instability of nonlinear Hamiltonian systems, with some applications to geometric optics, using KAM theory. (6) Other research started under this grant is still in progress.

One of the student supported by this grant has already received his Ph.D.; he won an NSF Postdoctoral Fellowship. He also received the Joaquin B. Diaz prize for the best thesis in mathematical sciences at RPI for 1995.

19960520 060

14. SUBJECT TERMS

15. NUMBER OF PAGES
5

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
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Final Report on grant F49620-92-J-0049

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February 20, 1996

Most of the main results obtained with the support of this grant are contained in the following publications. Some other new ideas and directions are still being developed and are not yet published. One of these is the geometrical role of curvature in high–frequency averaging, with applications to control by electromagnetic or acoustic fields.

1. A new randomness–generating mechanism in forced relaxation oscillations, a preprint.

2. V. Zharnitsky, The geometrical description of the nonlinear dynamics of a multiple pendulum, SIAM J. Applied Mathematics, Vol. 55, No. 6, pp. 1753–1763, 1995.

3. V. Zharnitsky, "Quasiperiodic motions in the billiard problem", to appear in Phys. Rev. Lett.

4. "Gyroscopic effects in a rotating sleeve hydrocyclone", Appl. Math. Lett. Vol. 6, No. 4, pp. 91-95, 1993.

5. "A theorem of Poinsot, path ordered integrals and parallel transport", Fields Institute Communications, Vol. 1, pp. 133-138.

6. "Geometric Phases in the Motion of Rigid Bodies", Arch. Rational Mech. Anal. 122 (1993) 213-229.

7. "A "bicycle wheel" proof of the Gauss-Bonnet theorem, dual cones and some mechanical manifestations of the Berry phase". Expo. Math. 12 (1994), 145-164

8. "Boundedness of solutions with quasiperiodic potentials", with E. Zehnder, SIAM J. Math. Anal. Vol. 26, No. 5, pp. 1233-1256, 1995.

9. "Geometric aspects of stability for Hill's equations", with H. Broer . Arch. Rational Mech. Anal. 131(1995) 225-240.

10. With S. Laederich, "The geometry of variational problems with non–holonomic constraints, ETH preprint.

11. with W. Weckesser, "Stabilization of the inverted linearized pendulum by high–frequency vibration", SIAM Review Vol. 37, No. 2, pp. 219-223, 1995.

12. "Universality in forced relaxation oscillations", to appear in Appl. Math. Lett.

13. "Composition of Rotations and Parallel Transport", Nonlinearity 9 (1996), 1-7 (to appear).

Below is a description of **selected** results contained in the above papers.

1 New phenomena in forced and coupled oscillations.

My main result in this area is the discovery of a large class of forced oscillators which exhibit deterministic randomness in a rigorous sense and in a physically meaningful way. This is the first time that this phenomenon has been demonstrated for differential equation which arise in physical applications, for instance, in electric circuits. Previously the so–called "chaos" has been demonstrated only for the physically insignificant set of initial conditions of zero measure, leaving a wide large gap between the experiment and the theory.

Another effect that I had discovered [12] is a normal form for relaxation oscillators: a large class of relaxation oscillators turns out to be much simpler than expected. In the limit such systems turn out to be piecewise linear!

2 Gyroscopic effects in a rotating sleeve hydrocyclone.

In this paper ([4])I gave an explanation of a puzzling phenomenon observed in the rotating sleeve hydrocyclone, a centrufugal device used to separate oil droplets suspended in water. The water is passed through a tube–shaped device, from one end to the other with the swirling motion, whose centrifugal effect causes the less dense oil particles to gravitate towards the axis of the tube; the oil core is then extracted though the openings at the two ends of the tube, with the purified water coming out of the opening at the perimeter of the end cap of the tube. The paper contains several observations, of which the main one is that the Taylor–Proudman effect plays the key role in explaining the observed behavior. (The effect consists in the apparent stiffening of the vortex lines if the vorticity of the fluid is close to a (large) constant.) An easy–to carry out experiment is proposed which would test the validity of the explanation, together with a possible improvement in the design of the device.

3 Geometry and stability in parametric resonance

The paper [9] gives a new observation about an old prolbem of parametric resonance in Hill's equations. It is well known that such equations have an infinite sequence of resonance gaps g_n. We show that as the forcing almplitude changes, the nth zone collapses n times! We prove this for an open set of systems and pose an interesting unsolved problem for the general case. Physically, this is a new observation in the quantum mechanical Stark effect.

4 A theorem of Poinsot, path ordered integrals and parallel transport.

In this paper [13] I address the so–called reconstruction problem: given the angular velocity of the rigid body as a function of time, what is the position of the body after time T? The discrete version of this question amounts to computing the product of a large number of orthogonal matrices. I found a curious formula for the products of orthogonal matrices (A referee's report on that paper is enclosed).

5 Other results

In the paper [8] we study stability of motions in a potential that depends quasiperiodically on time. In terms of geometrical optics this result implies the confinement for all time of a ray inside an optical fiber with a quasiperiodic index of refraction. A similar intrerpretation applies to the motion of charged particles in electromagnetic field with quasiperiodic time and space–dependence. This work requires a rather delicate application of KAM theory. The difficulty is in reducing the problem to that of a small perturbation of an integrable system.

The paper [6] gives some observations on the geometry of nonholonomic constraints; some of these observations are of interest in robotics.

The yet unpublished work (started in the Spring of 1995) shows a beautiful relationship between a certain curvature on the one hand and high–frequency averaging on the other. This observation gives a new insight into the effective potentials of Kapitsa. The ideas of this work have applications in electromagnetic and acoustic levitation and control.

6 Work by graduate students supported by this grant.

In the paper [2] V. Zharnitsky solved the problem considered earlier by N. Rott and others by numerical and asymptotic methods. The system in question is a double pendulum, or of any ocillating conservative 2 degree of freedom system near the 2:1 resonance. Zharnitsky describes the global behavior of the system, and in particular to explains why the phase drift vanishes for some initial conditions and doesn't for others. Zharnitsky gives a single compact geometrical picture which shows at a glance the breakdown of the phase space into solutions which are "locked in" and into the others which are not. He also describes the bifurcations that this picture undergoes as the parameters of the system vary. This paper gives a nice bridge between theory and application.

Zharnitsky received his Ph. D. degree this February. He was offered a postdoctoral position at Los Alamos. He was also awarded an NSF Postdoctoral Fellowship. He also received the Joaquin B. Diaz prize for the best thesis in mathematical sciences at RPI for 1995.

In the joint *expository* paper [11] with W. Weckesser. we give the simplest to date explanation of the well–known and well–studied phenomenon

of stabilization of the inverted pendulum whose suspension point undergoes vertical oscillations. The standard textbook treatments of this subject rely on calculations.

With partial support from this grant, Weckesser came up with a new idea which allowed him to prove linear stability of a large class of rotating systems, including the twirling chain. Many good mathematicians tried this problem without success.

Much of this work would have been impossible without support from this grant.